Abstract

This thesis presents new data structures for several orthogonal intersection searching problems. It also shows a lower bound for implicit priority queues supporting the decrease-key operation.

The orthogonal intersection searching problems are all studied from an upper-bound perspective and in the RAM model of computation. The 2-dimensional dynamic orthogonal range reporting problem is to maintain, under insertions and deletions, a set of points in the plane such that given a query rectangle with sides parallel to the coordinate axes, the set of points inside the rectangle can be reported. The problem naturally generalizes to any dimension larger than or equal to one. Similarly, the 2-dimensional dynamic segment reporting problem is to maintain, again under insertions and deletions, a set of horizontal line segments in the plane such that given a vertical segment the set of horizontal segments intersecting it, can be reported. Both problems can also be considered in the static case where updates are not allowed but where the objects (points or segments) are given in advance for preprocessing.

In Chapter 2, we give a solution to the 2-dimensional dynamic orthogonal range reporting problem and to the 2-dimensional dynamic orthogonal segment intersection reporting problem. Both these solutions support updates in time $O(\log n)$ and queries in time $O(\log n + k)$ where n is the number of objects (points or vertical line segments) and k is the size of the output. The structures use space $O(n \log n / \log \log n)$. By reduction to the dictionary problem the time usage of the structures is in some sence optimal if coordinates of objects can only be accessed by a comparison operation.

In Chapter 3, the orthogonal range reporting structure of Chapter 2 is improved to allow updates in time $O(\log^{\delta} n)$ for some (not any) constant $\delta < 1$ and queries in time $O(\log n / \log \log n + k)$. Here we need to assume a slightly different way of specifying coordinates to bypass the $\Omega(\log n)$ lower bound that exists if we can only compare coordinates. The space usage of the structure is $O(n \log^{\delta} n)$. The time usage of the structure is in some sense optimal in the cell probe model of computation by a lower bound of Alstrup et. al.. The structure is extended to arbitrary constant dimension $d \geq 2$ where it has update time $O(\log^{d+\delta-2} n)$, query time $O((\log n / \log \log n)^{d-1} + k)$ and space usage $O(n \log^{d+\delta-2} n)$.

In Chapter 4, we consider the one-dimensional variant of the dynamic orthogonal range reporting problem. Here we assume each coordinate is an integer which fits into a computer word of w bits. We show a whole range of tradeoffs between update time and query time where one of them is that we for any constant $\epsilon > 0$ can support updates in amortized time $O(w^{\epsilon})$ with high probability and queries in worst case constant time. We also give a structure for dynamic approximate range counting in one dimension. All the structures of Chapter 4 use linear space.

In Chapter 5, we consider two problems. The first is the static version of dominance reporting in any constant dimension $d \geq 3$. This problem is a variant of orthogonal range reporting where the kind of queries that can be asked are restricted. We show that for this problem it is possible to answer queries in time $O((\log n / \log \log n)^{d-2} + k)$ using space only $O(n(\log n / \log \log n)^{d-3})$. The next problem we consider is a static variant of range counting in any constant dimension $d \geq 2$ where the goal is not to report the points inside the query but to count the number of points.

We show that for this problem it is possible to answer queries in time $O((\log n/\log\log n)^{d-1})$ using space $O(n(\log n/\log\log n)^{d-2})$.

In Chapter 6, we consider a generalized variant of orthogonal range reporting called *colored* range reporting. Here we assume that objects (points or horizontal line segments) have a color and the objective is not to report objects but to report the different colors of the objects. We give a structure for the dynamic 1-dimensional version of this problem where the n points are supposed to be in an array of size n. The structure uses linear space and supports updates in time $O(\log\log n)$ with high probability and queries in worst case time $O(\log\log n + k)$. We use this result together with partial persistence to obtain improved upper bounds for the 2-dimensional static colored variants of both orthogonal range reporting and orthogonal segment reporting.

In Chapter 7, we give lower bounds for an implicit heap supporting the decrease-key operation. Our major result is a lower bound stating that unless delete is allowed to take very long time, then the amortized cost of decrease-key must be $\Omega(\log^* n)$. The lower bound is obtained by reducing the decrease-key operation to the *Absent Minded Usher's Problem* which we also introduce in this thesis.