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Abstract. An representation based on the singularity structure of the gradient magnitude over scale is used as the atoms in a space of images. This representation is summarized as a rooted tree. The generic transitions of the functional of the scale space images are analysed and listed for the scale parameter and one free parameter. A distance measure between images is deduced soly from these generic transistions. The singular transitions are translated into the language of the tree transitions such that one generic transitions corresponds to one unit edit operation of the tree structure. The distance between two images is the size of the smallest set of edit operations necessary to transform the corresponding tree representations into each other.

1 Introduction

The content of an image manifests itself at multiple, a prior unknown levels of scale or resolution. This has been addressed by the computer vision community in a principled fashion by the so-called scale space theories or multi scale schemes[9, 11]. Scale space theory ensures an image representation invariant to rotation, translation and scaling (or invariant to other groups of transformations[22]) and provides a regularization of the original image to a differentiable output which makes the vast toolbox of differential geometry available. In its simplest form, a scale space image is a continuum of increasingly blurred images also refered to as the Gaussian scale space due to the generating kernel. In this paper we will not consider the vast amount of non-linear formulations of scale space schemes.

This machinery has opened for the creation of a range of feature detectors (interest points) defined and detected by (semi-)algebraic expressions of differential derivatives possibly automatically tuned to the appropriate scale[16]. The framework is mathematically well founded in a principled way allowing for derivation and analysis of properties of the system [23, 3, 20]. Recent research has investigated the geometry of scale space images [2] for instance the trajectories of extrema[4]. The geometry of scale space images relates the details present at low scale (high resolution) to the coarse overall objects on the high scale (low resolution) [17] and offers the opportunity to analyse information over scale also denoted deep structure analysis.

Information of objects over a range of scales can be represented as graphs or trees in the algorithmic data structure sense. Such a scale space tree of features provides an invariant representation as above but also an representation which is invariant to minor changes in the configurations of the singularities in the scale space image more precisely invariant to a small local diffeomorphism. Hence it represents the topology of the singularity paths over scale. Preliminary investigations [24, 18, 12] show that it is feasible to construct scale space trees based on different kinds of singularities also for 3D data sets.

Distance measures between the scale space trees can be established to assess the distance between the content in two different images. Standard graph or tree matching algorithms do not per default provide good distance measure between scale space trees, simply because an atomic tree transform in the algorithmic sense can correspond to a large series of scale space image transformations and vice versa. Some schemes have to be developed for finding the appropriate atomic transformation from the image analysis point of view and find their algorithmic counterpart for implementation purposes.

For this, we propose the generic tree transformations to be deduced from the generic transitions of the singular paths. The necessity for only generic transitions is obvious. All imaginable transitions will give rise to a combinatoric explosion of possible transitions but according to singularity theory only the generic cases will occur in almost all cases (loosely spoken the non-generic cases occur with probability zero) and are very limited in number. The presence of non-generic structures [13, 10]can for instance indicated unlikely symmetry relativ to structure in all possible images.

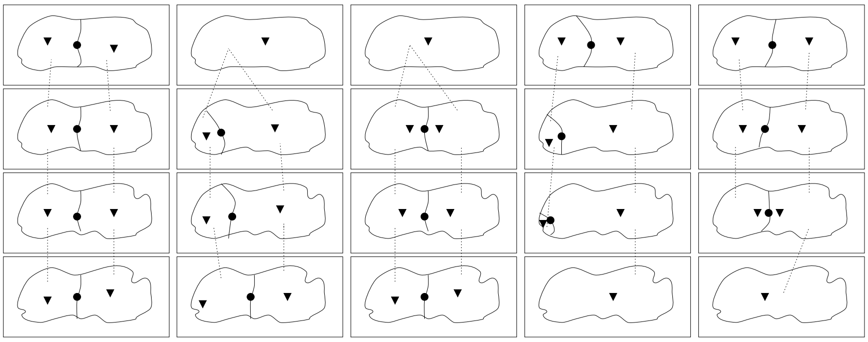
Genericity is always stated in relation to a base set of functions. In this paper it is the set of solutions to the heat equation.

The proposal is inspired by the successful line of work within shape analysis, specifically within medial representations/skeletonisations of shapes [5, 6] and symmetry set representation [5, 6, 14]. In this area, classical singularity theory has been applied and extended to determine the relation between geometric fiducial points on the outline of the shape and the central points in the medial representations. Next step has been to derive the generic transitions for the fiducial points for general warping of the shape and relate these results to the corresponding changes in the medial representation [7]. The counterpart in the scale space tree approach is to establish the generic transitions of a scale space image when the original image is changed. These transitions will be translated into the language of algorithmic tree transitions[19].

We present an extended annotated scalespace tree detected from the multiscale structure of the squared gradient magnitude. We will derive the list of possible transitions for the singularities of the gradient squared under the parameters of scale and an extra control parameter. These transitions will be use to deduce corresponding tree transitions which will form the basis for an image matching scheme.

2 A multiscale gradient magnitude tree

The gradient magnitude from a Gaussian scale space image has previously been suggested as the underlying representation for a semi-automatic segmentation [21, 15, 1]. For this, a tree structure was constructed based on the generic transitions of an image evolving under the heat equation, the complete list of these transitions was derived and analysed in previous work[20]. It was shown that the fold and the cusp catastrophes occur generically in Gaussian scale space for the squared gradient magnitude.



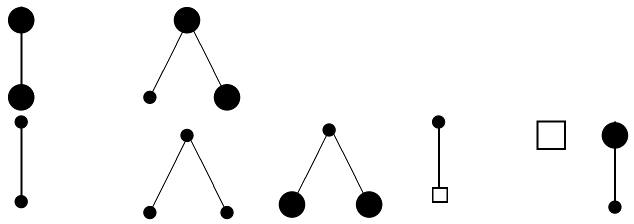
(a) No change — (b) Annihilation fold local — (c) Merging cusp global-to-local — (d) Creation fold local — (e) Splitting cusp local-to-global

Fig. 1. Generic events of the gradient magnitude. Scale increases upwards in the figure. Minima and saddles are symbolised with triangles and circles, respectively.

The possible transitions through the sampled scale space is shown schematic in figure 1. Also included in the figure are the dual regions to the minima namely the catchment basins or Voronoi areas. The events (annihilation, merging, creation, splitting) are named after the interaction between the saddle and the minimum (or minima). In the cases of annihilation (b) and merging (c) two minima and a saddle are reduced to one minimum, corresponding to the disappearing of a border between the two segments. The cases of creation (d) and splitting (e) are the reverse events where the emerging saddle corresponds to the appearing of a border between the segments (dual to the two minima). Hence, the edges in the tree are in all cases given by the saddle connecting the involved minima. A line from a segment to a segment indicates a edge.

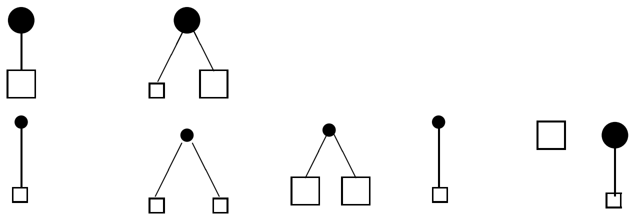
The annihilation or creation of one minimum with one saddle will always involve a non-global/local minimum. The merging will involve the joining of two global minima and saddle into one local minimum. The splitting event will destroy one local minimum and introduce one saddle and two global minima at a higher scale. These facts originate from the simple fact that global minima for the gradient magnitude squared correspond to singularities in the original image and the cusp event in the gradient magnitude image will correspond to fold event in the original image. Hence global minima in the gradient magnitude will always interact in pairs never alone.

This extra information will be used to annotated the tree structure with the type of the different minima. Nodes in the tree correspond to the minima of the squared gradient magnitude of the scale space image. Each level of the tree corresponds to a sampled scale in the scale space image. The nodes in the tree are connected according to derived possible transitions of the minima. The nodes are annotated as local or global minima. This more rich tree structure limits later on the amount of possible matches inbetween trees. That is, for a given part of the first tree less possible matches exist in the second tree being compared to the first one. Of course this also results in a more detailed list of possible transitions.



(a) No change (b) Annihilation fold (c) Merging cusp (d) Creation fold (e) Splitting cusp

Fig. 2. The tree syntax corresponding to the generic scale transitions in figure 1. Circles are internal nodes. Squares are leaves. Big symbols correspond to global minima and small symbols correspond to local.



(a) No change (b) Annihilation fold (c) Merging cusp (d) Creation fold (e) Splitting cusp

Fig. 3. The tree syntax as in figure 2 instantiated on the lowest level in the tree.

The full tree syntax is presented in figure 2. In the generic transitions of the singularities in figure 1 were not directly indicated the difference between transition with local and global minima. This is included in figure 2. Circles denoted internal nodes in a tree, small and big circles indicate respective local and global minima. In case (a) there is no change of either a local or global minimum. In case (b) a local minimum is annihilated and its corresponding node will be connected to its neighbour which is either a local or global minimum. In case (c) two global minima are merging into one local minimúm. In case (d) a local minimum is created somewhere in the tree. In case (e) a local minimum splittes into two global minima. Hence the local minima is linked to a global minima which will have a global leaf as neighbour. In this paper we only consider the framework of trees not graphs. Therefore in case (e) the local minimum is only connected to one of the global minima. The representation of a splitting as a child node with two parents would ruin the tree structure and introduce the more general graph structure. Such a representation has been discussed as a interesting and relevant alternative in previous publication by the author and others [8, 17].

Each of the cases and the subcases in figure 2 can occur on the lowest level (the root is in the top) of the tree. In such a case the lowest circles in each case will be transform to leaves and denoted with a square instead. This is illustrated in figure 3

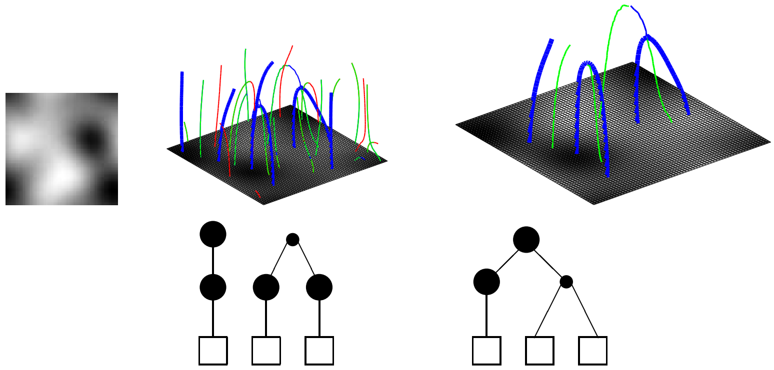


Fig. 4. Top left: An image, top middle: singularity paths for the image (red=maximum, green=saddle,fat blue=global minimum, thin blue=local minimum), top right: a subset of the paths. Bottom: the two groups in the subset of singularity paths correspond to two disjoint parts of the tree. Note that only the blue paths correspond directly to nodes and edges in the tree. Bottom left: the first group is a merging of two global minima into one local minima plus a path with no change. Bottom right: two global minima (leaves) merge into one local minima which annhilates and is connected to the neighbouring global minimum. Please note that the latter global minimum is for simplicity of the figure only depicted in the tree syntax and among all the paths in the middel not in the subset of paths shown to the top right.

In figure 4 is an example of an image and the corresponding singularity paths. For simplicity a subset of these has been selected and the corresponding tree structure for this subset is depicted. Note how the subtree are constructed by combine the subtrees from figure 2 and for the lowest level in tree the subtrees from figure 2 are used.

3 Tree transformations

When one image is warped into another this will of course also change the corresponding multi scale trees from one to another. Because the trees are deduced and build from the catastrophes in the singular paths the transitions of the trees will correspond to the sudden change of the paths known as higher order events in singularity theory or catastrophes (an abrut chagen of structure). In terms of singularity theory a family of multi scale images is a family of functions controlled by two parameters (scale and another one) from this the possible transitions can be analysed and the corresponding transition on the tree structure can be deduced.

In the following we provide an example of the connection between transitions of paths and tree transitions before giving the full list of tree transitions. In figure 5 is depicted a constructed image sequence. In figure 6 are the constructed frames plus their corresponding singular paths.

In figure 6 several series of transitions occur. In figure 7 only a subset is presented for clarity. The subset consists of two independent series of transitions. One in the background and one the foreground.

In the foreground series the connection between three global minima is changed. A “no change” path and a merge between two global minima interact and changes connections such that the “no change” path afterwards merges with middle global path and the far right path becomes a “no change” paths. Another phrasing would be that the middle

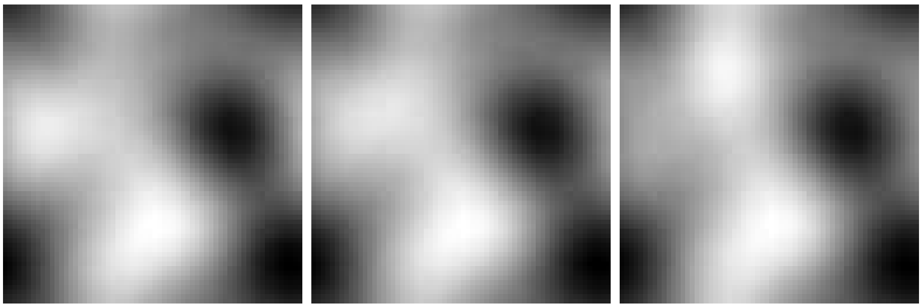


Fig. 5. A constructed image sequence made by taken a random image and adding a one-pixel size peak in three different locations and then blurr the sum. In left frame the peak is close the to left border of the image then moving along a straight line ending close to top part of the image in the last right frame.

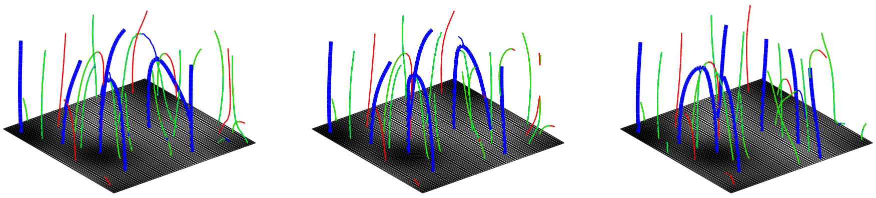


Fig. 6. Frames plus all singular paths. Red denotes maxima of gradient magnitude (G). Green denotes saddles of G. Thin blue is local minima. Fat blue is global minima.

global minima swaps its relation to its nearest neighbour; it swaps from being the detail of one structure to being a detail of the neighbouring structure.

The background series involves two changes first the annihilation in the top is resolved from the left frame to the middle frame, secondly the merge disappear from the middle frame to the right frame. In the presented scale range it correspond to an extended lifetime over scale of the involved structure. The first event the disappearing of the annihilation corresponds to the structure persists further over scale instead of becoming a detail of a larger structure. The second event has a similar interpretation since the global minima persists over a long scale range extend beyond the visualised levels of scale.

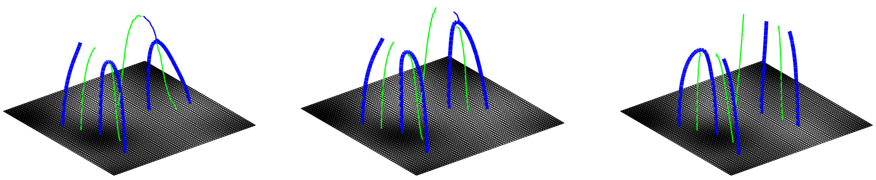


Fig. 7. A subset of the singular paths in fi gure 6. Two series of transitions are depicted. Green indicates saddles. Thin blue corresponds local minimum. Fat blue denotes global minima.

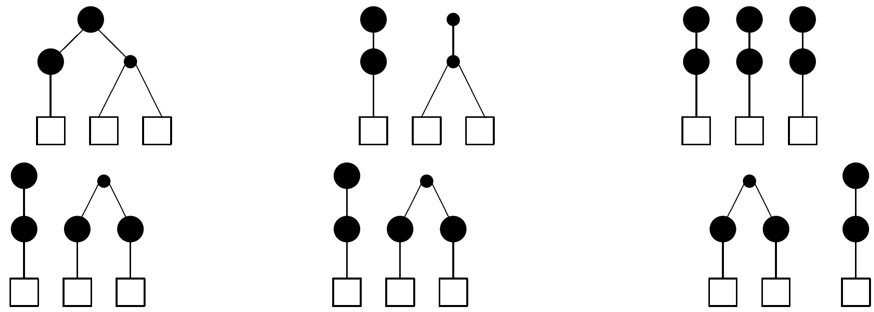


Fig. 8. The tree structures corresponding to the minima paths in figure 7. The top row corresponds the series of transitions in background of figure 7. The bottom row corresponds to the foreground series of transitions.

In figure 8 is shown the tree structure corresponding to the singular paths in figure 7. As is illustrated the minimal singular paths corresponds directly to the derived tree structure. Nodes correspond to the sampled scale levels and the edges in the tre is derive from catastrophe points on the singular paths.

3.1 Generic Tree transformations

Only a very limited number of local tree transformations is possible if one only consider transformations correspond to the generic transitions of the singular paths. In figure 9 is listed all the generic edit operations on the tree structures.

In the following section we will derive and explain how exactly these transitions are derived from the catastrophes.

4 Co-dimension one transitions in scaled families of the gradient magnitude

The co dimension one transitions are introduced by an extra control parameter such that the family of gaussian scale space images depends on two spatial variables, the scale parameter and the control parameter. One can think of the control parameter as describing a path through the underlying set of images. When traveling through the family of scale space images using this extra degree of freedom then for all most all image instances there will only be the usual catastrophes on the singular paths but for specific fixed values of the control parameter the singular paths will exhibit extra ordinary catastrophes (which are non-generic for a single scale space image) but are generic for a family. These are the transitions between two set of generic paths. These higher order catastrophes correspond to the collision of an extra singular path through a "ordinary" catastrophe point or the correspond to the resolving of a catastrophe or the shifting of it to a high or low location in scale.

In the following it can be useful to compare the transitions with the treesyntax in figure 2. An annihilation event for the gradient magnitude can be shifted towards higher or lower scale. This will correspond to the tree transition illustrated in figure 9 top row left.

A creation event can also be shifted in the scale direction which will result in the tree transition depicted in figure 9 middle row left.

A shift for the merge event will result in tree transition shown in figure 9 top row right. As in the other subfigures the top and bottom nodes stay fixed in the tree; hence the transition is fully depicted in the figures.

The splitting event can make shift in accordance with the illustration in figure 9 middle row right. Please note the difference to the middle row left where there is no restriction on the neighboring nodes.

The cusp catastrophe with two global minima and a saddle (corresponding to ordinary fold catastrophe for the image involving an extremum and a saddle) can collide with another global minimum path and swap the connectivity between the three involved global minima. This correspond to the transition in figure 9 bottom left.

The fold catastrophe can collide with another local minimum path (this will in the transition moment correspond to the cusp catastrophe for local minima). This will also result in the swap of the connectivity between the involved minima. This corresponds to the transition given in figure 9 bottom right.

This concludes the list of possible transitions and their counterpart in terms of tree transitions.

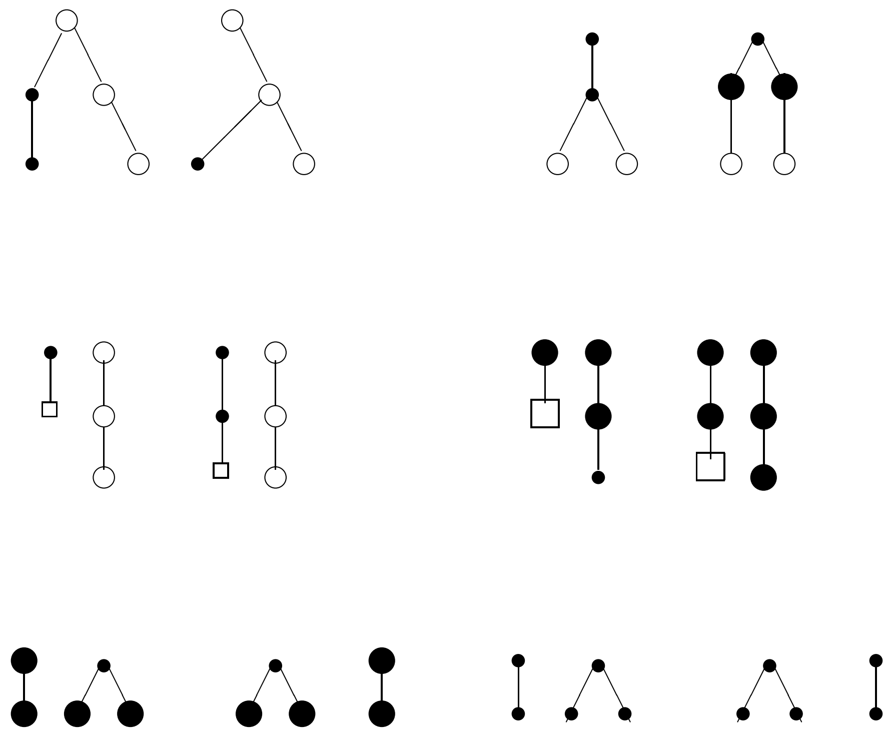


Fig. 9. The four types of tree transitions. The transitions can occur from left to right and from right to left. Open circles indicate either a local or global minimum annotation. These can be expanded according to the tree syntax provided in the previous sections.

5 Conclusion

An image representation based on multi scale singularity tree has been proposed. The syntax of the resulting multi scale tree has been presented. The possible transitions of the multi scale trees have been listed as the basis of image matching scheme based on edit operation distance of trees. In order to do this, the codimension one transitions of the scale space images have been derived and translated to tree transitions. It remains to apply this matching scheme to ensembles of real world images and evaluate its practical performance. This is the objective for future work.

References

1. Erik Dam, Peter Johansen, Ole Fogh Olsen, Andreas Thomsen, Tron Darvann, Andy B. Dobrzeniecki, Nuno V. Hermann, Noriyuki Kitai, Sven Kreiborg, Per Larsen, Martin Lillholm, and Mads Nielsen. Interactive multi-scale segmentation in clinical use. In *European Congress of Radiology 2000*, March 2000. Abstract and video accepted for CompuRAD.
2. James Damon. Local morse theory for solutions to the heat equation and gaussian blurring. *Journal of Differential Equations*, 115(2), January 1995.
3. James Damon. Properties of ridges and cores for two-dimensional images. *Journal of Mathematical Imaging and Vision*, 10:163–174, 1999.
4. Luc M J Florack and Arjan Kuijper. The topological structure of scale-space images. *Journal of Mathematical Imaging and Vision*, 12:65–79, 2000.
5. P. J. Giblin and B. B. Kimia. On the local form and transitions of symmetry sets, and medial axes, and shocks in 2D. In *Proc. of ICCV*, pages 385–391, Greece, September 1999. IEEE Computer Society.
6. P. J. Giblin and B. B. Kimia. On the local form of symmetry sets, and medial axes, and shocks in 3D. In *Proc. of CVPR*, pages 566–573. IEEE Computer Society, 2000.

7. P. J. Giblin and B. B. Kimia. Transitions of the 3D medial axis under a one-parameter family of deformations. In *ECCV*, pages 718–724, 2002.
8. Lewis D. Griffi n. *Descriptions of Image Structure*. PhD thesis, Uni. of London, 1995.
9. T. Iijima. Basic theory on normalization of a pattern (in case of typical one-dimensional pattern). *Bulletin of Electrotechnical Laboratory*, 26:368–388, 1962. (in Japanese).
10. Peter Johansen, Mads Nielsen, and Ole Fogh Olsen. Singular points in one-dimensional gaussian scale space. Journal of Mathematical Imaging and Vision, 2000.
11. Jan J. Koenderink. The structure of images. *Biological Cybernetics*, 50:363–370, 1984.
12. Arjan Kuijper and L.M.J. Florack. Using catastrophe theory to derive trees from images. *Journal of Mathematical Imaging and Vision*, 2004.
13. Arjan Kuijper and Luc Florack. The relevance of non-generic events in scale space models. *International Journal of Computer Vision*, 2004.
14. Arjan Kuijper, Ole Fogh Olsen, and Peter Giblin. Data structures from shapes using the symmetry set. In *SIAM conference on Imaging Science*, 2004.
15. M. Letteboer, O.F. Olsen, E. Dam, P. Willems, M. Viergever, and W.J. Niessen. Segmentation of tumors in mr brain images using an interactive multi-scale watershed algorithm. *Academic Radiology*, 2004.
16. Tony Lindeberg. *Scale–Space Theory in Computer Vision*. The Kluwer International Series in Engineering and Computer Science. Kluwer Academic Publishers, Boston, USA, 1994.
17. Ole Fogh Olsen. *Generic Image Structure*. PhD thesis, Department of Computer Science, University of Copenhagen, Universitetsparken 1, DK-2200 Copenhagen East, Denmark, March 2000. Technical Report DIKU-2000/04.
18. Ole Fogh Olsen. The scale structure of the gradient magnitude. Technical report, IT University of Copenhagen, Rued langgaards Vej 7, DK-2300 Copenhagen, Denmark, 2003. ITU Technical report : TR-2003-29.
19. Ole Fogh Olsen. Inducing edit distance from generic transitions. In *SIAM conference on imaging science*, 2004.
20. Ole Fogh Olsen and Mads Nielsen. Generic events for the gradient squared with application to multi-scale segmentation. In *Scale-Space Theory in Computer Vision, Proc. 1st International Conference*, volume 1252 of *Lecture Notes in Computer Science*, pages 101–112, Utrecht, The Netherlands, July 1997.
21. Ole Fogh Olsen and Mads Nielsen. Multi-scale gradient magnitude watershed segmentation. In *ICIAP'97 - 9th International Conference on Image Analysis and Processing*, volume 1310 of *Lecture Notes in Computer Science*, pages 6–13, Florence, Italy, September 1997.
22. Peter Olver, Guillermo Sapiro, and Allen Tannenbaum. Differential invariance signatures and flows in computer vision: a symmetry group approach. In Bart ter Haar Romeny, editor, *Geometry-Driven Diffusion in Computer Vision*, chapter 11. Kluwer Academic Publishers, 1994.
23. J.H. Rieger. Generic evolutions of edges on families of diffused greyvalue surfaces. *Journal of Mathematical Imaging and Vision*, 5(3):207–217, 1995.
24. K. Somchaipeng, J. Sporring, S. Kreiborg, and P. Johansen. Software for extracting 3d mssts. Technical report, EU project IST-2001-35443, Deliverable No. 8 http://www.itu.dk/Internet/sw1953.asp.